Research Journal of Recent Sciences ISSN 2277-2502
Vol. 3(**ISC-2013**), 9-11 (**2014**) *Res. J. Recent. Sci.*

Stochastic model for estimation of Fecundability in between two successive Live births (Closed Birth Interval)

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Available online at: www.isca.in, www.isca.me
Received 2nd December 2013, revised 7th February 2014, accepted 19th February 2014

Abstract

*Fertility as the positive force is responsible for the growth of human population. The researchers have given priority to understanding of the determinants of fertility through statistical methodologies. The stochastic models play an important role in estimation and interpretation of the fertility parameters. In this paper, stochastic model on successive live births has been derived for the estimation of fecundability based on assumptions of human reproductive process, indirectly incorporating socio, bio-demographic factors, taboos and use of contraceptive practices. In this model to describe the variation in the length of i^{th} order successive live births for female giving their $(i+1)^{th}$ birth in T years of married life after i^{th} birth with the realistic assumption that all the females not exposed to the risk of conception immediately after post-partum amenorrhea (PPA) termination due to some factors or contraceptive practices. In this derived model, fecundability (**λ**) has been considered to be constant over the study period. The duration of time from the point of termination of PPA to the state of exposure has been taken as random variable which follows exponential distribution. The maximum likelihood estimation technique has been used for the estimation of parameter (**λ**) through derived model.*

Keywords: Fecundability, birth interval, post partum amenorrhea (PPA), contraceptive practices.

Introduction

Fertility behavior is usually influenced by the action and the interaction of a number of complex factors. Within the physiological limits of human reproduction, it is determined by a multiplicity if biosocial and demographic factors. The role of differentials in fertility has been reported by a number of researchers[1-3]. Mathematical models are very appropriate tools and are widely used for better understanding of the phenomenon of the complex process of human reproduction. In other words, these models are useful in understanding of the action and interaction or inter relationship among various factors as well as for predicting the change in fertility behavior. Gini was the first in this area to initiate research in model building, by introducing the concept of fecundability[4]. In general, the biological variables have been accounted for and it assumed that factors such as socio, cultural, demographic etc. Sheps, Singh and others have given detailed discussions on the variables to be included in the model[5-6].

Some of the Main Biological Factors

Fecundability: It is defined as the probability that a non pregnant fecund woman will conceive in one unit of the time of the exposure to the risk of conception. The unit is taken as one month which is the length of a menstrual cycle.

Sterility: A female is said to be sterile if conception is impossible physiologically.

Foetal Wastage: A conception may not always result in a live birth. The outcome of the corresponding pregnancy may end in a spontaneous abortion, an induced abortion and still birth.

Non Susceptible Period: This is the sum of the two parts; first, gestation period and second the interval after its termination and before the resumption of the ovulation, which is the known as post partum amenorrhea (PPA) period.

There are two broad categories of the fertility model. First, the models which deal with the utilization of the data on point events like conception, live births to women in a specified period of time. The second type of models utilizes the data on interval between the consecutive events. Both type of models have own usefulness as well as limitations. The present paper is associated with second type of model. Various categories of birth intervals discussed so far in the literature are:

First Birth Interval: The interval between the marriages to first live birth. This interval gives the recent marital fertility performance.

Close Birth Interval: The interval between two successive live births. This gives the actual fertility performance in between two successive birth as well as impact of PPA and temporary separation and impact of family planning.

Open Birth Interval: The interval between the dates of birth of last child to the date of the survey. This provides the latest fertility performance.

Forward Birth Interval: The interval between survey date and date of next live birth posterior to survey date.

In the last decade's considerable attention has been given to analyze the data on closed birth interval or interval between successive live births. The main importance of the closed birth interval is due to inclusion of amenorrhoiec period, temporary separation due to social taboos or use of contraceptives. Bhattacharya et al. derived a probability model to interval between successive live births by taking different parametric form of risk function and assumed one to one correspondence between conception and a live birth[7]. Singh derived time dependent model for inter live birth interval with finite exposure period by taking into account intrauterine mortality and a distribution for the non- susceptible period[8]. Singh derived analytical models for human fertility behavior and their applications with the consideration of socio cultural factors[9]. Mturi studied the determinants of birth intervals on the Tanzanian women[10]. Rama Rao discussed birth interval and their implications of birth spacing strategies in Mozambique[11]. Recently, Singh et al. described the demographic and socio-economic determinants of birth interval dynamics[12]. Yadav et al. calculated parity progression ratios from open and close birth interval data[13].

Due to the socio cultural factors or contraceptive practices, some females not exposed to the risk of conception immediately after the termination of PPA. In this paper, stochastic model on successive live births has been derived for the estimation of fecundability based on assumptions of human reproductive process, indirectly incorporating socio, bio-demographic factors, taboos and use of contraceptive practices.

In this model to described the variation in the length of i^{th} order close birth interval for females giving their $(i+1)^{th}$ birth in T years of married life since i^{th} birth considering the fact that females exposed to the risk of conception at different points after the termination of PPA.

In this model the fecundability has been assumed to be constant over the study period and the duration of time from the point of termination of PPA to the state of exposure has been taken as random variable which follows on exponential distribution.

Model: Suppose a cohort of females observed for T period of time since i^{th} birth and all the birth intervals between i^{th} and $(i+1)^{th}$ birth which occur on or before T are obtained. The probability density function of the waiting time between i^{th} and $(i+1)^{th}$ birth is derived Assumption: i. The female has led married life throughout the period of observation. ii. Let h be the constant duration of non- susceptibility associated with each live birth comprised of gestation and the period of PPA. iii. The duration of non- susceptibility after the termination of PPA which is caused by some social factors or use of contraceptive practices be a non negative random variable. Let the female after termination of her PPA will enter into susceptible state in a small interval $(t, t+\Delta t)$ is $\mu\ \Delta t + 0\ \Delta t$; $\mu > 0$, $\Delta t > 0$ and $t > 0$. iv. Let the females who are susceptible to conception at time t will conceive in a small interval $(t, t+\Delta t)$ is $\lambda\ \Delta t + 0\ \Delta t$; $\lambda > 0$, $\Delta t > 0$ and $t > 0$. v. Let each conception result into a live birth.

For a female, interval between i^{th} and $(i+1)^{th}$ birth say T_i ($i \geq 1$) in the absence of risk of foetal wastage. This interval is the sum of four components. i. z: the duration of PPA. ii. y: duration of non- susceptible period caused by some social factors or use of contraception just after the termination of PPA. iii. x: waiting time from the date of entrance into the susceptible period to the first conception. iv. g: the gestation period.

Therefore, $T_i = z + y + x + g$
Under the above assumption the probability density function and the corresponding distribution function of the interval between i^{th} and $(i+1)^{th}$ birth say $f_i(t)$ and $F_i(t)$, will be as
$f_i(t) = [\mu\lambda/(\mu-\lambda)]\ [e^{-\lambda(t-h)} - e^{-\mu(t-h)}]$ if $i \geq 1$ and $t > h$

$F_i(t)= [1- \mu/(\mu-\lambda)\ e^{-\lambda(t-h)} + \lambda/(\mu-\lambda)\ e^{-\mu(t-h)}]$ if $i \geq 1$ and $t > h$

Now the probability density function and probability distribution function i^{th} and $(i+1)^{th}$ birth who have given their $(i+1)^{th}$ birth during the first T years after i^{th} birth will be given as $f^*(t) = f(t)/F(t)$ if $h < t < T$ [14].

According to above condition the probability density $f^*(t)$ and probability distribution function of present model defined as
$f^*(t) = [\mu\lambda/(\mu-\lambda)]\ [e^{-\lambda(t-h)} - e^{-\mu(t-h)}] / [1- \mu/(\mu-\lambda)\ e^{-\lambda(t-h)} + \lambda/(\mu-\lambda)\ e^{-\mu(t-h)}]$ if $h < t < T$

$F^*(t) = [1- \mu/(\mu-\lambda)\ e^{-\lambda(t-h)} + \lambda/(\mu-\lambda)\ e^{-\mu(t-h)}] / [1- \mu/(\mu-\lambda)\ e^{-\lambda(T-h)} + \lambda/(\mu-\lambda)\ e^{-\mu(T-h)}]$ if $h < t \leq T$

In the distribution derived so far it has been assumed that non susceptible period h associated to a live birth was constant for all female. But in practice it is empirically observed that the duration of PPA varies from female to female while it may be assumed to be a constant. If it is assumed that h takes value $h_1 < h_2 < \ldots\ldots < h_q$ with respective proportions of females b_1, b_2, ………, b_q. The probability density function and probability distribution function of the derived model can easily be obtained. The parameter λ and μ through method of maximum likelihood (MLE) with the help of large sample data are obtained.

Results

The derived model has been illustrated for a set of observed data relating to the interval between first and second birth in T years of married life after the first birth. The MLE estimate of the parameters λ and μ of the model are obtained to real data of rural Eastern Uttar Pradesh, India. The MLE of λ=1.3783 and μ= 2.7450. On the basis of Chi-Square goodness of fit the above derived model and also described the real situation satisfactorily. The estimate of risk parameter is higher and observed mean of first close birth interval 2.53 years.

Conclusion

The model seems to estimate the parameter satisfactorily. The goodness of fit of the model with the observed distribution of closed birth interval indicates that assumptions on which the model is developed are quite consistent. The high estimate of μ gives that almost all the females were exposed to the risk of conception within a year after termination of PPA. The role of socio-cultural and contraceptive practices for a very short time period after the termination of PPA may be due to age at marriage of couples or stabilization of family. In this derived model, assumptions are quite strong, it is hoped that some modifications of it may be describe the situation better. The inclusion of risk of fetal wastage might improve the applicability of the model.

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